Number Base Addition LESSON

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Objective

In this lesson you'll learn how to do simple addition in Base 2, 8, and 16.

It is essentially the same as Base 10 addition, just in a different base!

Review Base Ten Addition, #1

In Base 10 addition, you learned a very simple process.
Look at this problem:

$$12$$
$$\underline{+37}$$

First add the ones column, then the tens.

Review Base Ten Addition, #1

$$
\begin{array}{r}
12 \\
+37 \\
\hline
49
\end{array}
$$

The answer is 49…simple, right?

Review Base Ten Addition, #2

Now look at this problem:

     13
+37

When you add the ones column values, the result of 10 EQUALS the base value of 10, so you have to CARRY a 1.

Review Base Ten Addition, #2

$$
\begin{array}{r}
\textcolor{red}{1} \\
13 \\
+37 \\
\hline
0
\end{array}
$$

When a carry is made, you essentially divide by 10 (the base) to determine what value to carry, and mod by 10 to determine what value to leave behind.

Review Base Ten Addition, #2

1

13

+37
___

0

3 plus 7 is 10

10 divided by 10 is 1 (carry)

10 mod 10 is 0 (leave)

Review Base Ten Addition, #2

$$\begin{array}{r} 1 \\ 13 \\ +37 \\ \hline 50 \end{array}$$

Answer is 50

Review Base Ten Addition, #3

Here's a third example:

$$\begin{array}{r} 16 \\ +\underline{37} \end{array}$$

When you add the ones column values, the result of 13 EXCEEDS the base value of 10, so CARRY a 1.

Review Base Ten Addition, #3

16
+37

6 plus 7 is 13
13 divided by 10 is 1 (carry)
13 mod 10 is 3 (leave)

Review Base Ten Addition, #3

1
16
+37
——
53

Answer is 53

O(N)CS Lessons

Review Base Ten Addition, #4

And finally, a fourth example:

$$\begin{array}{r} 76 \\ +35 \\ \hline \end{array}$$

The ones column result of 11 EXCEEDS the base value of 10, and you CARRY a 1.

Review Base Ten Addition, #4

1
76
+35
———
1

6 plus 5 is 11

11 divided by 10 is 1 (carry)

11 mod 10 is 1 (leave)

Review Base Ten Addition, #4

1

76

+35

1

1+7+3 is 6 plus 5, which equals 11

11 divided by 10 is 1 (carry)

11 mod 10 is 1 (leave)

Review Base Ten Addition, #4

1

76
+35
111

Answer is 111, base 10

Base Eight Addition, #1

Now here is an example in base eight:

$$\begin{array}{r} 12 \\ +34 \\ \hline \end{array}$$

When you add the ones column values, the answer is 6, and the second column answer is 4.

Base Eight Addition. #1

$$
\begin{array}{r}
12 \\
+34 \\
\hline
48
\end{array}
$$

Answer is 48, base eight

You say, "four eight base eight", *not "forty-eight"*

The phrase "forty-eight" is meant for base ten only.

Base Eight Addition, #2

Now look at this problem:

$$\begin{array}{r} 14 \\ +34 \\ \hline \end{array}$$

When you add the ones column values, the result of 8 EQUALS the base value of 8, and you have to CARRY a one.

Base Eight Addition, #2

14
+34

Again you divide by 8 (the base) to determine what value to carry, and mod by 8 to determine what value to leave behind.

Base Eight Addition, #2

$$
\begin{array}{r}
{\color{red}1} \\
14 \\
+34 \\
\hline
0
\end{array}
$$

4 plus 4 is 8

8 divided by 8 is 1 (carry)

8 mod 8 is 0 (leave)

Base Eight Addition, #2

1

```
   1
  14
 +34
 ----
  50
```

Answer is "five zero, base eight"!
Looks strange, but it is correct!

Base Eight Addition, #3

Here's a third example:

$$\begin{array}{r} 16 \\ +37 \\ \hline \end{array}$$

When you add the ones column values, the result of 13 EXCEEDS the base value of 8, and you have to CARRY a one.

1

16

+37
___

5

6 plus 7 is 13

13 divided by 8 is 1 (carry)

13 mod 8 is 5 (leave)

Base Eight Addition, #3

1

16

+37
___

55

Answer is 55, base eight.

Base Eight Addition, #4

And a fourth example:

$$76$$
$$\underline{+35}$$

The ones column result of 11 EXCEEDS the base value of 8, …CARRY a one.

Base Eight Addition, #4

$$\begin{array}{r} \textcolor{red}{1} \\ 76 \\ +35 \\ \hline 3 \end{array}$$

6 plus 5 is 11

11 divided by 8 is 1 (carry)

11 mod 8 is 3 (leave)

$$\begin{array}{r} 1 \\ 76 \\ +35 \\ \hline 33 \end{array}$$

1+7+3 is 6 plus 5 is 11

11 divided by 8 is 1 (carry)

11 mod 8 is 3 (leave)

Base Eight Addition, #4

1

```
  76
+35
───
133
```

Answer is 133, base 8

Base Two Addition, #1

Base Two Addition is quite interesting, but also fairly simple.

Since the only counting digits in base two are the values 0 and 1, there are only a few situations you have to learn.

Base Two Addition, #1

We'll start simple:

$$1$$
$$\underline{+1}$$
$$=10 \text{ ("one zero, base two")}$$

This looks strange, but the same process applies.

Base Two Addition, #1

$$1$$
$$\underline{+1}$$
$$=\ 10$$

Since 1 + 1 is 2, this EQUALS the base value of 2, which means you carry the "div" answer and leave the "mod" answer

Base Two Addition, #1

        1
      +1
   = 10

2 / 2 = 1 (carry)

2 % 2 = 0 (leave)

That's it!

Base Two Addition, #2

Here's another:

```
     10
    +11
   ─────
 =  101
```

Can you figure it out?

Base Two Addition, #2

$$10$$
$$\underline{+11}$$
$$= \ 101$$

In the ones column, 1 + 0 is 1.

In the second column, 1+1 is 2, or 10 in base 2

Base Two Addition, #3

And another:

```
  101101
 +110011
```

=

Can you figure it out?

Base Two Addition, #3

Step by step…

$$
\begin{array}{r}
\color{red}{1} \\
101101 \\
+110011 \\
\hline
= 0
\end{array}
$$

Base Two Addition, #3

Step by step…

$$
\begin{array}{r}
\textcolor{red}{1} \\
101101 \\
+110011 \\
\hline
=00
\end{array}
$$

Base Two Addition, #3

Step by step…

$$\begin{array}{r} \color{red}{1} \\ 101101 \\ +110011 \\ \hline =000 \end{array}$$

Base Two Addition, #3

Step by step…

$$
\begin{array}{r}
\textcolor{red}{1} \\
101101 \\
+110011 \\
\hline
= \quad 0000
\end{array}
$$

Base Two Addition, #3

Step by step…

$$
\begin{array}{r}
\color{red}{1} \\
101101 \\
+110011 \\
\hline
= \ 00000
\end{array}
$$

Since 1+1+1 is 3, carry 1 and leave 1

Base Two Addition, #3

Step by step…

$$\begin{array}{r} \color{red}{1} \\ 101101 \\ +110011 \\ \hline =1100000 \end{array}$$
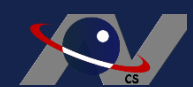
All done!

Base Sixteen, Example #1

In base sixteen, remember the digits are 0-9, then A-F, representing the values 0-15

Here's an example:

$$\begin{array}{r} 29 \\ +12 \\ \hline \end{array}$$

```
   29
 +12
 = 3B, base 16
```

2 + 9 is 11, which is B in base sixteen

2+1 is 3, so the answer is 3B

Base Sixteen, Example #2

1

```
   A9
 +47
= F0, base 16
```

9+7 is 16, equal to the base, so carry 1 and leave 0

1 + A(10) + 4 is 15, which is F

Base Sixteen, Example #3

11

```
  D6
 +7C
= 152, base 16
```

6+C(12) = 18, carry 1, leave 2

1+D(13)+7 = 21, carry 1, leave 5

Base Sixteen, Example #4

11

EF

+2D

= 11C, base 16

F(15) + D(13) = 28, carry 1, leave C(12)

1 + E(14) + 2 = 17, carry 1, leave 1

Exercises

Now try these exercises
1. $1_2 + 1_2 =$
2. $7_8 + 6_8 =$
3. $F_{16} + F_{16} =$
4. $5_8 + 5_8 =$
5. $9_{16} + B_{16} =$
6. $C_{16} + D_{16} =$

Exercises

7. $3_8 + 4_8 =$

8. $F_{16} + 2_{16} =$

9. $10_2 + 10_2 =$

10. $1_2 + 1011_2 =$

11. $10_2 + 110_2 =$

12. $216_8 + 364_8 =$

13. $777_8 + 3_8 =$

Exercises

14. $ACE_{16} + BAD_{16} =$

15. $234_{16} + 975_{16} =$

16. $42_{16} + F_{16} + 876_{16} =$

ANSWERS (JUMBLED)

7   11   12   10   14   15   19

1E   100   602   BA9 8C7   1000

1002   1100   167B