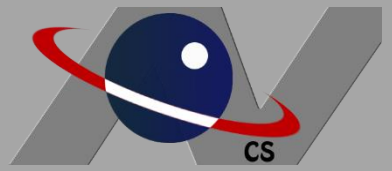
Number Bases
LESSON ONE

By John Owen



Objective

In this lesson you'll learn about different Number Bases, specifically about those used by the computer

Those include:

- Base Two – binary
- Base Eight – octal
- Base Sixteen – hexadecimal

Base Ten

First let’s talk about good ‘ole base ten, the Decimal number system which humans use and you have been working with for centuries.

It’s called base ten because…?

Come on, you know this right?

Base Ten

If you said, “because it has ten counting digits, 0,1,2,3,4,5,6,7,8, and 9”, you are right!

To count in base ten, you go from 0 to 9, then do combinations of two digits starting with 10 all the way to 99.

Base Ten

After 99 comes three-digit combinations from 100 to 999, etc.

This combination system is true for any base you use.

The only difference is how many digits you have before you go to the next combination

Base Two

To count in base two, which only has 0 and 1 as counting digits, you count 0,1, then switch to two digit combinations, 10,11, then to three digit combos, 100, 101,110,111, then four digit, 1000, _____,_____, …, 1111

Base Three

To count in base three, which has 0, 1, and 2 as counting digits, you count 0,1,2, then switch to two digit combinations, 10,11, 12, 20, 21, 22, then to three digit combos, 100, 101,102, 110,111, 112, etc…

Base Eight

Jumping to base eight, what are the counting digits?

Can you count correctly, switching to two-digit combinations, then three-digit combos correctly?

Base Eight

Here is the base eight counting sequence

0,1,2,3,4,5,6,7,10,11,12,13,…77

100,101,102,103,104,105,106,107

110,111, etc.

Base Sixteen

Now for one that's a bit strange.

Base Sixteen, also known as *hexadecimal,* was especially created by computer scientists to help in low-level programming, like machine language and assembly language.

Base Sixteen

To count in base sixteen, you need 16 counting digits.

To get sixteen counting digits, you use 0-9, but you need six more…so it was decided to use A,B,C,D,E, and F.

Base Sixteen

The symbol **A** represents the value 10, **B** is 11, **C** is 12, **D** is 13, **E** is 14, and **F** is 15.

Here's the single digit sequence for base sixteen:

0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Base Sixteen

Then the two-digit combos:

10,11,12,…19,1A,1B,1C,1D,1E,1F,20,21,22,…2D,2E,2F,30,31,…FF

Get the picture?

Base conversion

To convert from base ten to another base, such as base two, eight, or sixteen, is an important skill for computer scientists and programmers.

The next section shows how to do this.

Base Ten to Base Two

Let’s take the value 27 and convert it into base 2.

Here’s the process:

- Divide 27 by 2
- The answer is 13, remainder 1
- Divide 13 by 2
- Answer is 6, remainder 1

Base Ten to Base Two

Continue until the answer is 1.

6 divided by 2 = 3, remainder 0

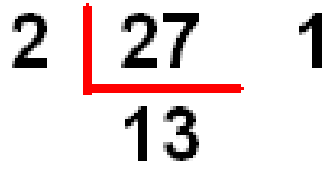
3 divided by 2 = 1, remainder 1

Now take the last answer, 1, and all of the remainders in reverse order, and put them together…11011

27 base 10 = 11011 base two

Base Ten to Base Two

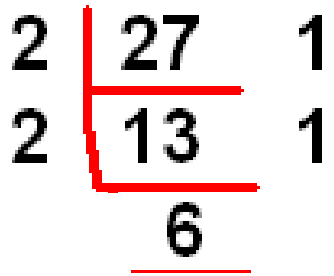
Here's an easy way to do it on paper



27 divided by 2 = 13, R 1

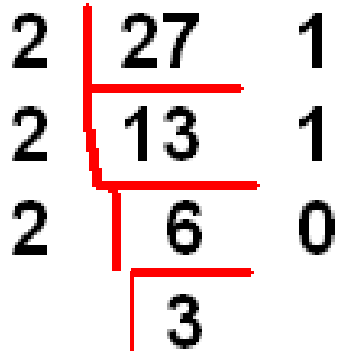
Base Ten to Base Two

13 / 2 = 6, R 1



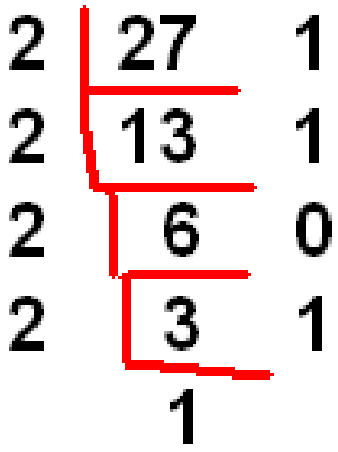
Base Ten to Base Two

6 / 2 = 3, R 0



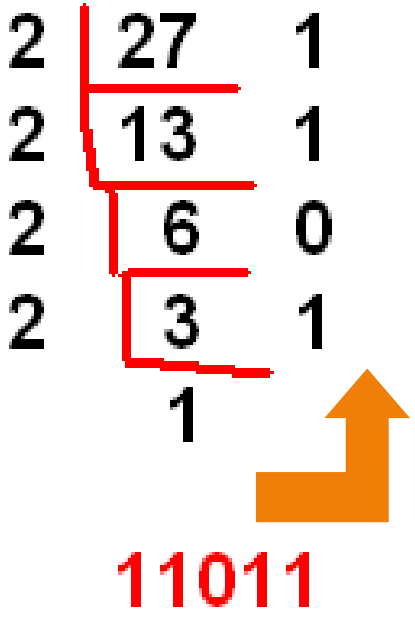
Base Ten to Base Two

3 / 2 = 1, R 1



Base Ten to Base Two

Stop when the answer is less than the divisor, and write the answer from the bottom to the top.



Exercises

Now try a few yourself (see last slide for answers):

1. 16_{10} = _____$_2$
2. 47_{10} = _____$_2$
3. 145_{10} = _____$_2$
4. 31_{10} = _____$_2$
5. 32_{10} = _____$_2$

Base Ten to Base Eight

Let's again take the value 27 and convert it into base 8.

Same process:

Divide 27 by 8

The answer is 3, remainder 3

Stop! You can't divide anymore because the answer is less than 8

Base Ten to Base Eight

The last answer was 3, and the only remainder was 3, so the base eight value is 33, base 8.

Base Ten to Base Eight

Use the same method on paper

27 divided by 8 = 3, R 3

27, base 10 = 33, base 8

Exercises

Now try the same values for base eight.

6. 16_{10} = _____$_8$
7. 47_{10} = _____$_8$
8. 145_{10} = _____$_8$
9. 31_{10} = _____$_8$
10. 32_{10} = _____$_8$

Base Ten to Base Sixteen

Finally we'll convert 27 into base 16.

Divide 27 by 16

The answer is 1, remainder 11

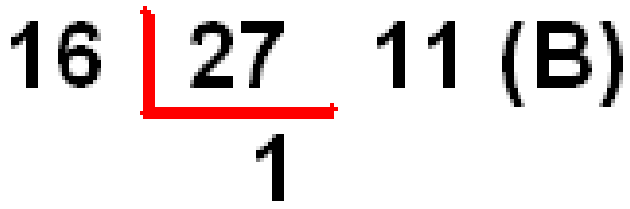
Stop! You can't divide anymore because the answer is less than 16

Base Ten to Base Sixteen

The last answer was 1, and the only remainder was 11, which in base 16 is the letter B, so the base sixteen value is 1B, base 16.

Base Ten to Base Sixteen

Again, the same method on paper



27 divided by 16 = 1, R 1

27, base 10 = 1B, base 1

Exercises

And now try base sixteen!

$11.16_{10} =$ _____$_{16}$

$12.47_{10} =$ _____$_{16}$

$13.145_{10} =$ _____$_{16}$

$14.31_{10} =$ _____$_{16}$

$15.32_{10} =$ _____$_{16}$

Conclusion

Now you should know how to count in different bases how to convert from

- Base ten to base 2
- Base ten to base 8
- Base ten to base 16

Here are the answers to the exercises, in jumbled order

10 1F 20 20 2F 37 40 57 91 221

10000 11111 101111 100000 10010001