Number Bases
LESSON TWO

By John Owen

CS

O(N)CS Lessons

Objective

In the last lesson you learned about different Number Bases used by the computer, which were

- Base Two – binary
- Base Eight – octal
- Base Sixteen – hexadecimal

Base Conversion

You also learned how to convert from the decimal (base ten) system to each of the new bases…binary, octal, and hexadecimal.

Other conversions

Now you will learn other conversions among these four number systems, specifically:

- **Binary to Decimal**
- **Octal to Decimal**
- **Hexadecimal to Decimal**

Other conversions

As well as

- **Binary to Octal**
- **Octal to Binary**
- **Binary to Hexadecimal**
- **Hexadecimal to Binary**

Other conversions

And finally

- **Octal to Hexadecimal**
- **Hexadecimal to Octal**

Binary to Decimal

Each binary digit in a binary number has a place value.

In the number 11<u>1</u>, base 2, the digit farthest to the right is in the "ones" place, like the base ten system, and is worth 1.

Technically this is the 2^0 place.

Binary to Decimal

The 2nd digit from the right, 111, is in the "twos" place, which could be called the "base" place, and is worth 2.

Technically this is the 2^1 place.

In base ten, this would be the "tens" place and would be worth 10.

Binary to Decimal

The 3rd digit from the right, 111, is in the “fours” place, or the “base squared” place, and is worth 4.

Technically this is the 2^2 place.

In base ten, this would be the “hundreds” place and would be worth 100.

Binary to Decimal

The total value of this binary number, 111, is 4+2+1, or seven.

In base ten, 111 would be worth 100 + 10 + 1, or one-hundred eleven.

Binary to Decimal

Can you figure the decimal values for these binary values? See the answers on the next slide.

11

101

110

1111

11011

Binary to Decimal

Here are the answers:

11 is 3 in base ten

101 is 5

110 is 6

1111 is 15

11011 is 27

Octal to Decimal

Octal digits have place values based on the value 8.

In the number 111, base 8, the digit farthest to the right is in the "ones" place and is worth 1.

Technically this is the 8^0 place.

Octal to Decimal

The 2nd digit from the right, 1<u>1</u>1, is in the “eights” place, the “base” place, and is worth 8.

Technically this is the 8^1 place.

Octal to Decimal

The 3rd digit from the right, 111, is in the “sixty-fours” place, the “base squared” place, and is worth 64.

Technically this is the 8^2 place.

Octal to Decimal

The total value of this octal number, 111, is 64+8+1, or seventy-three.

Octal to Decimal

Can you calculate the base 10 value for these octal values?

21

156

270

1164

2105

Octal to Decimal

Here are the answers:

21 is 17 in base 10

156 is 110

270 is 184

1164 is 628

2105 is 1093

Hexadecimal to Decimal

Hexadecimal digits have place values base on the value 16.

In the number 111, base 16, the digit farthest to the right is in the “ones” place and is worth 1.

Technically this is the 16^0 place.

Hexadecimal to Decimal

The 2nd digit from the right, 1<u>1</u>1, is in the "sixteens" place, the "base" place, and is worth 16.

Technically this is the 16^1 place.

Hexadecimal to Decimal

The 3rd digit from the right, 111, is in the “two hundred fifty-six” place, the “base squared” place, and is worth 256.

Technically this is the 16^2 place.

Hexadecimal to Decimal

The total value of this hexadecimal number, 111, is 256+16+1, or two hundred seventy-three.

Hexadecimal to Decimal

Can you figure the decimal equivalents for these hexadecimal values?

2A

15F

A7C

11BE

A10D

Hexadecimal to Decimal

Here are the answers:

2A is 42 in base 10

15F is 351

A7C is 2684

11BE is 4542

A10D is 41229

Binary to Octal

The conversion between binary and octal is quite simple.

Since 2 to the power of 3 equals 8, it takes 3 base 2 digits to combine to make a base 8 digit.

Binary to Octal

000 base 2 equals 0 base 8

$001_2 = 1_8$

$010_2 = 2_8$

$011_2 = 3_8$

$100_2 = 4_8$

$101_2 = 5_8$

$110_2 = 6_8$

$111_2 = 7_8$

Binary to Octal

What if you have more than three binary digits, like 110011?

Just separate the digits into groups of three from the right, then convert each group into the corresponding base 8 digit.

110 011 base 2 = 63 base 8

Binary to Octal

Try these:

111100

100101

111001

1100101

Hint: when the leftmost group has fewer than three digits, fill with zeroes from the left:

1100101 = 1 100 101 = 001 100 101

110011101

Binary to Octal

The answers are:

- $111100_2 = 74_8$
- $100101_2 = 45_8$
- $111001_2 = 71_8$
- $1100101_2 = 145_8$
- $110011101_2 = 635_8$

Binary to Hexadecimal

The conversion between binary and hexadecimal is equally simple.

Since 2 to the power of 4 equals 16, it takes 4 base 2 digits to combine to make a base 16 digit.

Binary to Hexadecimal

0000 base 2 equals 0 base 8

$0001_2 = 1_{16}$

$0010_2 = 2_{16}$

$0011_2 = 3_{16}$

$0100_2 = 4_{16}$

$0101_2 = 5_{16}$

$0110_2 = 6_{16}$

$0111_2 = 7_{16}$

$1000_2 = 8_{16}$

$1001_2 = 9_{16}$

$1010_2 = A_{16}$

$1011_2 = B_{16}$

$1100_2 = C_{16}$

$1101_2 = D_{16}$

$1110_2 = E_{16}$

$111_2 = F_{16}$

Binary to Hexadecimal

If you have more than four binary digits, like 11010111, again separate the digits into groups of four from the right, then convert each group into the corresponding base 16 digit.

1101 0111 base 2 = D7 base 16

Binary to Hexadecimal

Try these:

- 11011100
- 10110101
- 10011001
- 110110101
  - Hint: when the leftmost group has fewer than four digits, fill with zeroes on the left:
  - 110110101 = 1 1011 0101 = 0001 1011 0101
- 1101001011101

Binary to Hexadecimal

The answers are:

- $11011100_2 = DC_{16}$
- $10110101_2 = B5_{16}$
- $10011001_2 = 99_{16}$
- $110110101_2 = 1B5_{16}$
- $1\ 1010\ 0101\ 1101_2 = 1A5D_{16}$

Octal to Binary

Converting from Octal to Binary is just the inverse of Binary to Octal.

For each octal digit, translate it into the equivalent three-digit binary group.

For example, 45 base 8 equals 100 101 base 2

Hexadecimal to Binary

Converting from Hexadecimal to Binary is the inverse of Binary to Hexadecimal.

For each "hex" digit, translate it into the equivalent four-digit binary group.

For example, 45 base 16 equals 0100 0101 base 2

Octal and Hexadecimal to Binary Exercises

Convert each of these to binary:

- 63_8
- 123_{16}
- 75_8
- $A2D_{16}$
- 21_8
- $3FF_{16}$

Octal and Hexadecimal to Binary Exercises

The answers are:

- $63_8 = 110011_2$
- $123_{16} = 100100011_2$ (drop leading 0s)
- $75_8 = 111101_2$
- $A2D_{16} = 101000101101_2$
- $21_8 = 10001_2$
- $3FF_{16} = 1111111111_2$

Hexadecimal to Octal

Converting from Hexadecimal to Octal is a three-step process.

- First convert from “hex” to binary
- Regroup the bits from groups of four into groups of three.
- Then convert to an octal number.

Hexadecimal to Octal

For example:

$4A3_{16}$

$= \underline{0100}\ \underline{1010}\ \underline{0011}_2$

$= \underline{010}\ \underline{010}\ \underline{100}\ \underline{011}_2$

$= 2243_8$

Octal to Hexadecimal

Converting from Octal to Hexadecimal is a similar three-step process.

- First convert from octal to binary
- Regroup the bits from groups of three into groups of four.
- Then convert to an hex number.

Hexadecimal to Octal

For example:

371_8

$= \underline{011}\ \underline{111}\ \underline{001}_2$

$= \underline{1111}\ \underline{1001}_2$

$= F9_8$

Octal/Hexadecimal Practice

Convert each of these:

- 63_8 = ____________$_{16}$
- 123_{16} = ____________$_8$
- 75_8 = ____________$_{16}$
- $A2D_{16}$ = ____________$_8$
- 21_8 = ____________$_{16}$
- $3FF_{16}$ = ____________$_8$

Octal/Hexadecimal Practice

The answers are

- $63_8 = 33_{16}$
- $123_{16} = 443_8$
- $75_8 = 3D_{16}$
- $A2D_{16} = 5055_8$
- $21_8 = 11_{16}$
- $3FF_{16} = 1777_8$

Number Base Conversion Summary

Now you know twelve different number base conversions among the four different bases (2,8,10, and 16)

With practice you will be able to do these quickly and accurately, to the point of doing many of them in your head!

Practice

Now it is time to practice.

Go to the Number Base Exercises to find some excellent practice problems.

Good luck and have fun!